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Remarkable historic timber roofs. Knowledge and conservation practice.
PART 2 - Investigation, analysis, and interventions

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THE ROOFING STRUCTURES OF THE GOTHIC AGE IN FRANCE

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Abstract

The structural solution called *chevrons formant ferme* is a typical invention of the French Gothic age. In this paper, the structural functioning of such kind of structures is considered and analyzed, referring to the two original different structures of the roof of Notre-Dame of Paris, destroyed by the fire of 2019. The results show the incredible skill of the builders of the Middle Ages in designing very effective timber structures and how these structures were conceived to respond to criteria of different nature.

Keywords

Timber structures, Structural analysis, Design reconstruction, Gothic architecture.

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1. INTRODUCTION

The unexpected birth and rapid explosion of what we call, from the Renaissance, Gothic architecture, is an unprecedented architectural phenomenon that appeared in the North of France during the 12th century, so huge and wide that, according to Gimpel [1], during less than three centuries, the French builders carried and used more stones than the Egyptians during the whole period of their civilization. The typical Gothic construction is the cathedral that, beyond the peculiarities of the Gothic structural elements, like flying buttresses, pointed arches, and rib vaults, is characterized by its dimensions: a Gothic cathedral is a high and large stone building.

What is often forgotten in the description and architectural studies of the Gothic cathedrals is that these buildings were covered by a timber structure. Though not as apparent and visible as the stone body of a cathedral, these timber constructions, in French, the *charpentes*, are impressive constructions, often having huge dimensions. This fact, along with other constraints of different natures, detailed below, forced the carpenters of the Middle Ages to invent innovative structural solutions, peculiar to this period and geographical area, that time proved to be very effective.

The true invention of the carpenters of the Gothic period is what in French is called the *charpentes à chevrons formant ferme*. This expression, whose translation could be “a carpentry with rafters forming common frames”, indicates a particular structural solution that appeared in the French region around Paris during the first part of the 13th century.

Though this kind of structure has been studied extensively in the past by historians of architecture, rather curiously, no serious and complete structural studies of these constructions seem to exist in the literature. This fact has probably contributed to creating misunderstandings and false ideas about the structural functioning of these structures.

A detailed study of two roofing structures with *chevrons formant ferme* has been done recently on the two original *charpentes* of the choir and of the nave that covered Notre-Dame of Paris and were destroyed by the fire that occurred on April the 15th, 2019 [2].

The objective of the present paper is to try to give a reconstruction of the structural ideas of the Gothic builders, using to this end the results of the cited paper, i.e., considering the destroyed roof of Notre-Dame as a representative paradigm of a *chevrons formant ferme* roofing structure. The indications given by the structural calculations show some facts describing clearly the statics of the timber structure and, rather likely, the structural thought of the builders. From these facts, it is also evident that some ideas carried on by architecture historians are inaccurate. All these points are discussed below.

Finally, this research aims to go beyond the mere descriptive analyses done so far and try to shed light on its real static behavior, how, presumably, it was thought by its ancient master-builders. In some way, it is an attempt to retrace the constructional thinking of the Gothics, their ideas in designing their *charpentes*, and to check whether or not some of the more common ideas on this matter are sound.

The paper is so structured: first, a description of a *chevrons formant ferme* structure is given, with some examples. Then, the structural analysis results on the destroyed *charpentes* of Notre-Dame are recalled and analyzed. On the basis of these results, we try, on one side, to refute or confirm some ideas of the past about this kind of structure and then to try to respond to the most delicate question: why the carpenters of the gothic age invented such a sophisticated and innovative structure?

2. DESCRIPTION OF A *CHEVRONS FORMANT FERME* ROOFING STRUCTURE

A covering structure with *chevrons formant ferme* is basically composed of four main structural elements, see Fig. 1: the main frame, in French *ferme principale* or *chevron maître*, the common frames (*fermes secondaires* or *fermettes* or *chevrons*), the bracing system (*contreventement*) and the wall plates (*sablières*). All these elements work together to carry the vertical (dead load) and horizontal (wind loads) actions. This modular unit, composed of the main frames and a few common frames, is then repeated to form the whole roofing structure. Usually, the *charpente*, so constituted, was covered by a wooden decking (the *voligeage*) and by tiles or lead plates. The correspondence between English and French technical terms is given in Tab. 1.

Of course, the static scheme of the main and common frame, as well as of the bracing system, can change, each roofing structure being a peculiar case, but all of them share the same fundamental characteristics of having a main frame and some common frames that are placed at a short distance (say, of the order of 80÷100 cm) and by the absence of purlins. The rafters forming the common frames are not supported by purlins but are a part of the whole structure. The fundamental difference between the main and common frames is the absence, in these last, of the tie-beam: only collar ties are present in the common frames. This is a key point of the structural functioning of *chevrons formant ferme charpentes*. The reason for this choice made by the carpenters of the Middle Ages is analyzed below. Still, since now, it should be kept in mind that these choices (the absence of the tie-beam in the common frames and that of the purlins, with the consequence that the rafters are not carried but self-supporting) completely characterize this kind of structure: the rafters of the common frames actually form, with their collar ties, a (secondary) frame, whence the name of *charpente à chevrons formant ferme*. The whole system is then typically characterized by rafters (of the main and common frames) that are close together, which allows posing the wooden decking (the *voligeage*) directly on the structure, unlike in the structures composed of main frames, purlins, and rafters, see, e.g. [3, 4], that as a consequence have a higher global thickness of the covering structure.

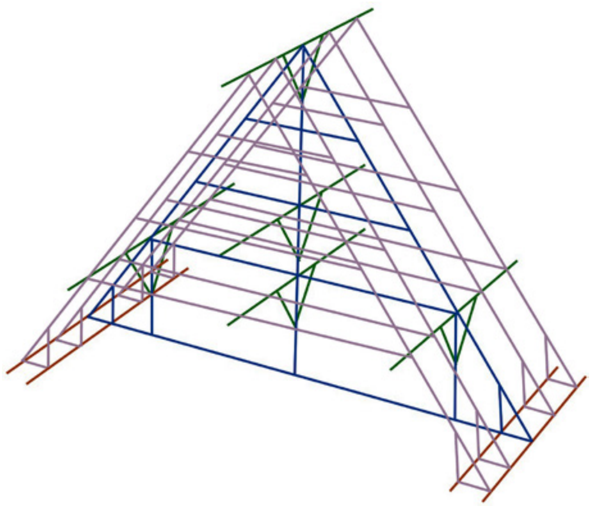

Fig. 1. *The modular unit of a charpente with chevrons formant ferme; in blue, the main frame (chevron maître); in grey, the common frames (fermettes); in green, the bracing system and in brown, the wall plates (sablières).*

Fig. 2. *Views of the combles: (a) in the choir (image source: [7]); (b) in the nave (image source: [8]); (c) in the transept (photo by the Author).*

This structural organization of the *charpente* gives it a three-dimensional structural functioning, which is not the case for the *charpentes* with main frames, purlins, and rafters. This is a fundamental aspect of the *charpentes à chevrons formant ferme*, which is detailed below. Before, a brief description of the *charpente* of Notre-Dame , used as a paradigm for the structural analysis, is given in the next section.

2.1. THE ANCIENT *CHARPENTES* OF NOTRE-DAME

The roof of Notre-Dame of Paris that burnt in the fire of April the 15th, 2019, was composed of three distinct *charpentes*, built at different periods ([5, 6] and Fig. 2): the choir *charpente*, made after 1220, probably from 1225 to 1230; the nave *charpente*, slightly subsequent, presumably built from 1230 to 1240; the transept *charpente*, entirely rebuilt during the restoration work of Lassus and Viollet-le-Duc after 1843, along with the spire and the first frames of the nave and choir nearby the spire. The *charpente* built in the 19th century was of the type with main frames, purlins, and rafters and will not be considered here.

The whole timber structure (also named the *combles* in French) had an overall length of 115.6 m, was 13 m wide, and was 9.75 m high. The two *charpentes* of the nave and choir were both à *chevrons formant ferme* but with different schemes. This fact, considered in detail below, suggests an evolution in the constructive thinking of the ancient carpenters [6, 8]. The longitudinal scheme of the *combles* is presented in Fig. 3a. The three parts of the *charpente* are clearly indicated as well as the notation of the main frames, as usually adopted. The longitudinal structure relying together the frames is schematically presented too. The parts of the *charpente* object of this analysis are those between frames FC4 and FC9 for the choir, and between FN1 and FN11, for the nave. They correspond to the regular part of the medieval *charpente*, the rest being the part of the structure constituting the apse of the choir that Lassus and Viollet-le-Duc reconstructed during the 19th century. Above the frames was a wooden decking, the voligeage, which was supporting the lead plates, nailed on it.

French term	English term
Charpente	Timber structure
Combles	Roofing structure
Ferme principale, chevron maître	Main frame
Ferme secondaire, fermette, chevron	Common frame
Panne faîtière	Ridge beam
Poinçon	King post, crown post
Suspente	Queen post
Poteau	Post
Entrait	Tie-beam
Faux entrait	Collar tie
Arbalétrier	Rafter
Faux arbalétrier	Secondary rafter
Jambette	Hammer post, ashlar piece
Aisselier	Brace, wind brace
Blochet	Hammerbeam
Sablières	Wall plate
Lierne	Girt
Jambe de force	Bracket
Contreventement	Bracing
Voligeage	Wooden decking
Passerelle	Catwalk
Mur gouttereau	Guttering wall
Console, corbeau	Cantilever, corbel

Tab. 1. *Correspondence between the French and English technical terms for the components of a charpente.*

In the choir of Notre-Dame , the modular unit was composed of one main frame and four common frames, spaced ~82 cm, for a whole length of ~4.1 m. In the nave, this distance has been reduced to ~3.5 m, with frames spaced ~75 cm. The main frames of the choir and nave of Notre-Dame , as well as the model of a *fermette*, which is sensibly the same in the two cases, are shown in Fig. 3b-d. These schemes have been reconstructed using mainly [9], who made the first dendrochronological analysis of the charpente in her MSc thesis, and [7]. In the same figure, the French names for the different pieces of the structure are also indicated. Unlike the choir's *charpente*, relatively homogeneous from FC4 to FC9, the nave's *charpente* presents minor differences in some frames. This is probably due to maintenance operations, done or not done, during the centuries. Here, being interested in analyzing the static functioning of the structure, we will consider just the modular structural unit around FN7, considered the most representative frame for the nave's *charpente*.

Though Dubu [10] said that the *charpente* was made with chestnut wood, it is certain that it was realized with the wood of oak trees, while the *voligeage* was made of fir wood, cf. [9], [8]. The number of trees employed for the structure was vast, so the *charpente* was called *la forêt*, the forest. According to F. Épaud, who

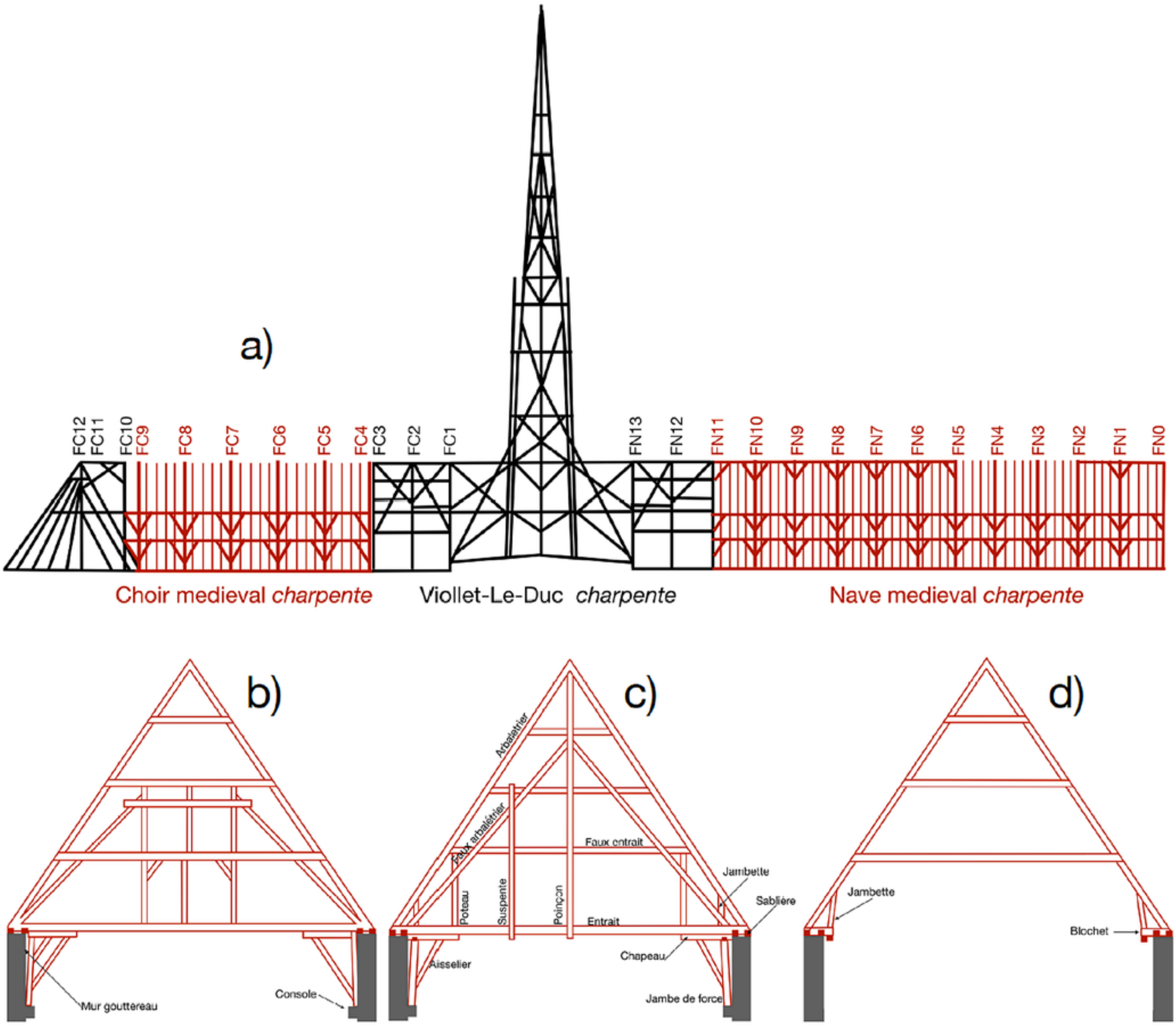

Fig. 3. *Schemes of the charpentes of Notre-Dame: (a) longitudinal scheme; the parts of the structure studied in this paper are in red; thick lines indicate the main frame and the longitudinal bracing system, while thin lines represent the common frames; (b) scheme of a main frame of the choir; (c) scheme of a main frame of the nave; (d) scheme of a common frame.*

has deeply studied the timber structures of the Middle Ages, it was composed of the wood of about 1000 oaks, almost all of them with a diameter of ~25 to 30 cm and 12 m high, a small part with a diameter of ~50 cm and 15 m high, corresponding to about 3 hectares of forest.

2.2. BRIEF HISTORICAL ACCOUNT OF THE NOTRE-DAME *CHARPENTES*

The first choir's *charpente* was probably finished before 1182 ([11], page 16; [12]) when the choir was consecrated. Subsequently, a new *charpente* was erected: on the choir between 1225 and 1230 and in the nave between 1230 and 1240 [5]. These new *charpentes* reused some timber beams of the original roof; in fact, several pieces of them showed unused mortices or *mi-bois* notches, a clear sign of reuse [11].

The reconstruction of the *charpente* was the consequence of a set of changes made to the cathedral during its construction. In particular, the guttering wall, i.e., the upper part of the clerestory, was raised about 2.70 m above its original height, a fact that had several consequences on the structure of the new *charpente*, as discussed below.

The reasons for this reconstruction are not well known, and historians still debate on this point: according to [13], the changes were done just for a matter of style, while [14] suggest that they were made to improve the structural response. Whatever the reasons for these modifications on the still unfinished cathedral, they are significant, do not concern the charpente uniquely, and are still a matter of historical debate today, cf. [12–15]. What is important for the purposes of the present study is that these transformations necessarily forced the carpenters to adopt a structural scheme different from the previous one. This is a crucial point to be analyzed below.

3. STRUCTURAL ANALYSIS OF THE *CHARPENTES À CHEVRONS FORMANT FERME* OF NOTRE-DAME

3.1. THE MECHANICAL MODELS

The *charpentes* of the choir and the nave are studied through a Finite element (FE) analysis. Each of the two structures is modeled as a truss, i.e., as an assembly of elastic rods pinned at the ends and each intermediate intersection with another beam. This choice is motivated by, on the one hand, the great uncertainties that exist in the ancient *charpentes*, where the gaps between the timber struts can be rather crucial because of the long drying of the wood pieces (as shown by the research of F. Épaud [16], the *charpentes* of the 12th and 13th centuries were realized with green wood), which implies a practically null couple at the joints, see also [17]. On the other hand, this assumption does not substantially affect the general structural functioning of the *charpente*, which is the study's objective.

The scheme of the main frames of the two *charpentes* and the common frames are shown in Fig. 4. The support points are denoted by the labels S1 to S10; all of them are modeled as frictionless unilateral supports, i.e., able

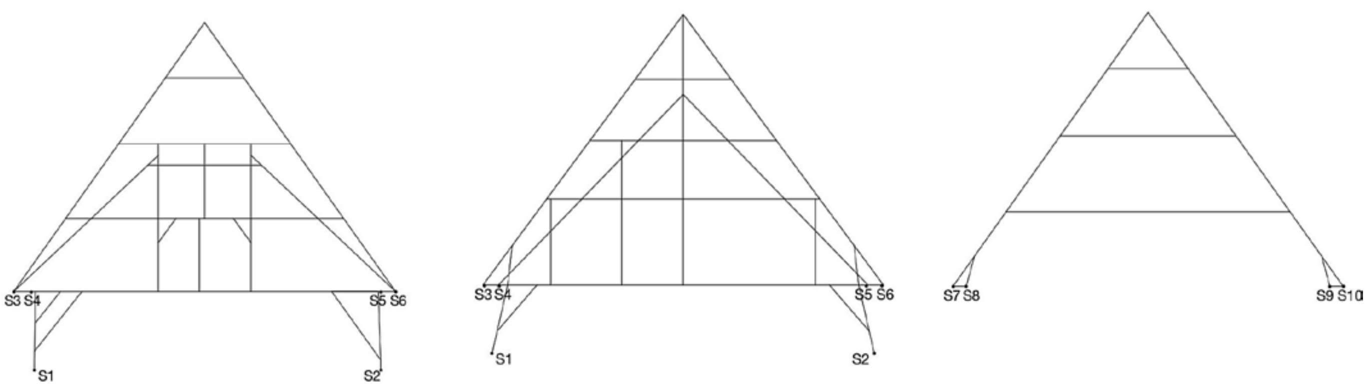

Fig. 4. *From left to right: schemes of the main frames of the choir, the nave, and the fermettes. The supports are indicated by letters S1 to S10.*

to exert only a purely contact reaction. The question of frictionless contact is examined below in section 3.5. In particular, the support points S3 to S10 can exert only upward vertical forces, while in S1 and S2, two unilateral reactions are exerted, an inward horizontal one and a vertical upward one. It is important to notice that as a consequence of the deformations produced by the loads, some of the support points can detach from their footing. The two structural units, the choir and the nave, are modeled through three-dimensional structural schemes shown in Fig. 5. The *sablières* are wall plates running from one main frame to another; they are just posed onto the top of the guttering walls and transmit the horizontal forces from the common to the main frames. They are hence bent in the horizontal plane. Each beam of the FE model is modeled as an Euler-Bernoulli rod. The boundary conditions imposed on the points at the ends of the *liernes* and *sablières* specify the continuity of displacements and rotations with the corresponding elements of the adjacent structural units. In this way, the simulation done on a singular structural unit represents the global structural response of the *charpente* for each one of its parts, in the assumption of uniform loading all over the *charpente*, which is the case for the own weight and, at least to a first approximation, for the wind action.

The dimensions of the beams composing the *charpente* are reported in Tab. 2; the minimum diameter d_{min} of a trunk to obtain the corresponding cross section is also indicated. As observed in [8], most parts of the *charpentes* can be obtained by trunks with a moderate diameter, less than ~30÷35 cm. A thickness of 2 cm for the *voligeage*, made of fir wood (density ~500 kg/m^3), has been considered. For the catwalk, the mass has been evaluated to ~240 kg for each structural unit of both the *charpentes*. The global volumes and masses of wood for each structural unit are summarized in Tab. 3.

The data above show that the quantity of wood is practically the same for the two *charpentes*, though the total mass per unit length is ~19% greater for the nave's *charpente*. In consideration of these data, what can be said is that the change of the structural scheme was not dictated by economical issues. The results of the structural analysis, shown below, suggest another possible reason: the carpenters of the 13th century probably searched for a better structural response. In fact, the main frame of the nave has an improved mechanical behavior than that of the choir; in short, it needs less wood to obtain the same stiffness: the nave's main frame is lighter than the choir's one. However, the nave's *charpente* unit is heavier than that of the choir

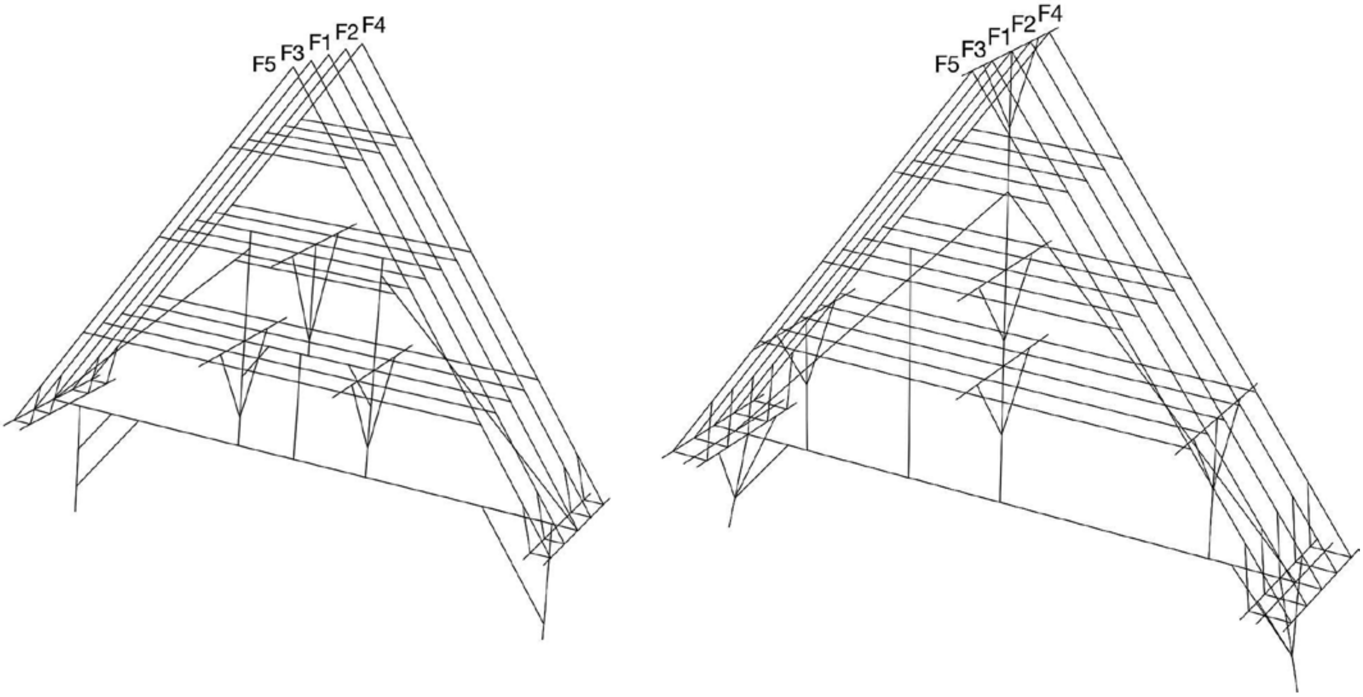

Fig. 5. *Schemes of modular structural units of the choir, left, and nave, right.*

because of the greater weight of the bracing system and of the common frames of the nave.

3.2. MATERIAL CHARACTERISTICS

The mechanical characteristics of oak wood are not constant; they change with the state of the wood, green or dry, and with the plants: as a matter of fact, a certain degree of uncertainty is unavoidable. In order to fix the necessary material parameters, the scientific publication of CIRAD [18] and the European norm EN338 [19] have been used. However, these data are not sufficient to completely characterize a cylindrically orthotropic material like oak wood [20]. In particular, the data taken from EN338 correspond to the wood class D50. Considering this insufficient data, the wood has been modeled as an isotropic material with Young's modulus E = 12500 MPa and a Poisson's ratio ν equal to 0.25. Because the structure is almost exclusively solicited by axial forces and bending moments, this choice does not substantially affect the way the structure works. For what concerns the density ρ, the value of 710 kg/m^3, generally accepted for dry oak wood, has been taken.

	b [cm]	h [cm]	d_{min} [cm]	A [cm^2]	J_1 [cm^4]	J_2 [cm^4]	J_0 [cm^4]	μ [kg/m]
Choir's *charpente*								
Entrait	30	35	46.1	1,050	107,188	78,750	185,938	74.550
1st *faux entrait, liernes*	13	27	30.0	351	21,323	4,943	26,267	24.921
2nd *faux entrait*	17	19.5	25.9	332	10,504	7,984	18,488	23.537
3rd *faux entrait*	15	23	27.5	345	15,209	6,469	21,678	24.495
4th *faux entrait*	15	19	24.2	285	8,574	5,344	13,918	20.235
Arbalétriers	18	19	26.2	342	10,289	9,234	19,523	24.282
Faux arbalétriers	28	17	32.8	476	11,464	31,099	42,562	33.796
Poteaux	19	15	24.2	285	5,344	8,574	13,918	20.235
Poteau central haut	14	14	19.8	196	3,201	3,201	6,403	13.916
Aisseliers faux entrait	14	17	22.0	238	5,732	3,887	9,619	16.898
Jambe gauche, jambettes	18	23	29.2	414	18,251	11,178	29,429	29.394
Aiss. j. gauche and *liernes*	14	18	22.8	252	6,804	4,116	10,920	17.892
Jambe droite	30	19	35.5	570	17,148	42,750	59,898	40.470
Aisselier jambe droite	30	18	35.0	540	14,580	40,500	55,080	38.340
Blochet	15	15	21.2	225	4,219	4,219	8,438	15.975
Sablières	19	14	23.6	266	4,345	8,002	12,347	18.886
Nave's *charpente*								
Entrait	26	29	38.9	754	52,843	42,475	95,318	53.534
Faux entraits	17	24	29.4	408	19,584	9,826	29,410	28.968
Arbalétriers	16	25.5	30.1	408	22,109	8,704	30,813	28.968
Faux arbalétriers	17	19	25.5	323	9,717	7,779	17,496	22.933
Poinçon	23.5	18.5	29.9	435	12,399	20,008	32,407	30.867
Suspente	12	12	17.0	288	3,456	3,456	6,912	20.448
Poteaux	17	20	26.2	340	11,333	8,188	19,522	24.140
Jambettes	15	16	21.9	240	5,120	4,500	9,620	17.040
Liernes	15	18	23.4	270	7,290	5,063	12,353	19.170
Aisseliers and *blochets*	15	15	21.2	225	4,219	4,219	8,438	15.975
Jambe de force	20	15	25.0	300	5,625	10,000	15,625	21.300
Chevrons secondaires	17	24	29.4	408	19,584	9,826	29,410	28.968
Sablières	22	14	26.1	308	5,031	12,423	17,453	21.868

Tab. 2. *Dimensions of the wood beams, as deduced from [7]; for each section, b is the width, h its height, d_{min} is the minimum diameter of the trunk, A the area, J_1 and J_2 the moments of inertia, J_0 the polar moment of inertia and μ the linear density of mass.*

3.3. LOADING CONDITIONS

Two loading conditions have been considered: own weight and own weight plus the wind. Following the European norm EUROCODE 1 [21], the wind action has been modeled as a static load, orthogonal to the surface, and distributed on the windward (overpressure) and leeward (suction) sides of the roof. The actions applied to the *charpente* are sketched in Fig. 6, and the values of the loads are detailed in Tab. 4.

	Choir		Nave	
	Mass [kg]	Volume [m³]	Mass [kg]	Volume [m³]
Ferme principale	3,168	4.46	2,920	4.11
Fermette	1,050	1.48	1,220	1.72
Contreventement and *sablières*	856	1.21	1,190	1.68
Total for the structure	5,074	7.15	5,330	7.51
Voligeage	920	1.84	766	1.53
Passerelle	240	0.34	240	0.34
Total for the structural unit	6,234	9.33	6,336	9.38
Total per unit length	1,520	2.27	1,810	2.68

Tab. 3. *Global quantities of wood for the choir and nave structural units.*

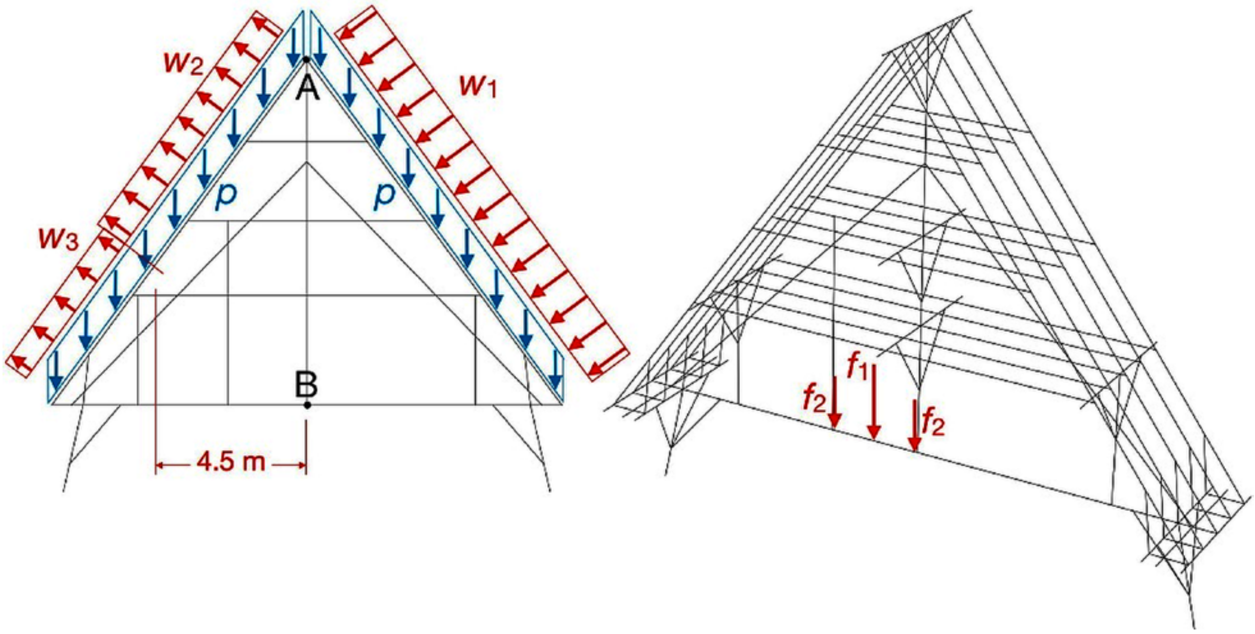

Fig. 6. *Scheme of the actions on the charpente; p: lead tiles and voligeage load; w_1, w_2, w_3: wind load; f_1, f_2: the load of the catwalk.*

	w_1	w_2	w_3	p_0	p_1	p	f_1	f_2
	[N/m]						[N]	
Choir	480.1	351.4	269.4	80.4	479.0	559.4	947	700
Nave	410.0	300.0	230.0	68.7	408.8	477.5	1,037	668

Tab. 4. *Loads on the charpente; p_0 is the linear load of the voligeage, p_1 that of the lead tiles, and p their sum; for the other symbols, refer to Fig. 6 (distributed loads are computed for a 1m wide strip of the roof).*

3.4. RESULTS OF THE NUMERICAL SIMULATIONS

The results of the FE simulations on the two *charpentes* are presented below. For understanding the structural functioning of the structure, especially in view of a critical analysis of the existing literature on the subject, introduced further, the following outputs of the numerical simulations have been considered: the distribution of the reaction forces, the displacements, the stresses, the support conditions, and in particular the role played by friction contact, and a modal analysis used to have an appraisal of the global stiffness of the structure. All these aspects are presented hereafter.

3.4.1. REACTION FORCES

The reaction forces at all the support points, S1 to S6 for the main frame F1, S7 to S10 for the secondary frames F2, F3 and F4, F5 (cf. Figs. 4 and 5), are detailed in Tab. 5 for the two *charpentes* and loading conditions. These reactions are also represented in Fig. 7. Observing these data and figures, the following remarks can be made:

- the distribution of the reactions for the loading condition OW (own weight) is almost, but not exactly, symmetric between the South and North sides; this is the consequence of small asymmetries in the main frames structure;
- still, for the OW loading condition, some of the contact forces are null, e.g., for the supports S7 and S10 of the choir's *fermettes*, or S7 for the F4 and F4 *fermettes* and S9 for the F2 and F3 *fermettes* of the nave's *charpente*;
- the vertical reactions in S1 and S2, i.e., at the level of the corbels supporting the *jambes*, are far less than that absorbed by nodes S3 to S6, on the guttering wall's top, for the choir's *charpente*, while it is higher in the nave, due to the different arrangement of the structure;
- for the loading condition OW+W (Own Weight + Wind), the *jambes* play a major role and strongly affect the distribution of the reactions; in particular, the set of supporting nodes changes: in the choir, nodes S3 on the North side are more charged, while nodes S4, S5, S8, and S10 are inactive; nodes S6, S7 and S9 are also active, but with a reaction force far below that of S3 and node S1 is charged only horizontally, to entirely absorb the wind thrust, while S2 is charged in the vertical direction, as an effect of the roof's slope;
- in the nave, nodes S2, S4, S6, S8 and S10 are inactive, while S1, S3, S5, S7 and S9 active; to remark the high reaction at node S5, consequence of the structural scheme;
- the slope of the roof, ~55°, ensures a stabilizing moment of the wind force distribution w_1 on the windward side, Fig. 6, which explains the positive reaction in nodes S2, S6, and S9 of the choir and the high reaction at node S5 of the nave's *charpente*;
- for both the *charpentes* and load conditions, the distribution of the reactions on the top of the guttering walls is far from being uniform: the largest part of the load is transmitted by the main frames, which play a fundamental role in the structural functioning of the *charpente*.

In Tab. 5, the resultant of the reactions are also indicated. This allows an estimation of the global loads and weights, which are summarized in Tab. 6.

3.4.2. DISPLACEMENT FIELD

The deformation of the modular structural units of the two *charpentes* is represented in Fig. 8; the values of the displacements of nodes A and B in Fig. 6 are shown in Tab. 7. In all the cases, the magnitude of the displacements is minimal. To remark that for the nave's *charpente* submitted to only vertical loads, the vertical displacement of point B, in correspondence with the middle of the tie-beam, is only 56% of the same displacement for the choir's structure. Moreover, the horizontal displacement of point A, the frame's top, of the nave is 44% of that of the choir. These data allow assessing the difference in stiffness of the nave and choir's *charpentes*; see section 3.6.

Frame	Node	Force [N]	Choir		Nave	
			OW	OW+W	OW	OW+W
F1	S1	Rx	2,472	3,7020	9,383	30,260
		Ry	5,084	0	30,220	69,940
	S2	Rx	-2,472	0	-9,383	0
		Ry	4,934	12,310	31,100	0
	S3	Rx	0	0	0	0
		Ry	29,830	51,980	12,090	5,984
	S4	Rx	0	0	0	0
		Ry	2,481	0	19,900	0
	S5	Rx	0	0	0	0
		Ry	2,322	0	15,450	45,600
	S6	Rx	0	0	0	0
		Ry	30,520	7,106	14,780	0
F2, F3	S7	Rx			0	0
		Ry	0	15,630	2,206	724
	S8	Rx	0	0	0	0
		Ry	10,230	0	453	0
	S9	Rx	0	0	0	0
		Ry	10,190	13,000	0	4,621
	S10	Rx	0	0	0	0
		Ry	0	0	2,873	0
F4, F5	S7	Rx	0	0	0	0
		Ry	0	1,540	0	2,736
	S8	Rx	0	0	0	0
		Ry	6,892	0	2,046	0
	S9	Rx	0	0	0	0
		Ry	6,883	8,362	1,587	4,287
	S10	Rx	0	0	0	0
		Ry	0	0	43	0
Resultants	Rx		0	37,020	0	30,260
	Ry		143,561	148,460	141,956	146,260
	Ry	South side	71,922	62,140	70,336	63,416
	Ry	North side	71,639	86,320	71,620	82,844
	Ry	on walls' top only	133,543	136,150	80,636	76,320
	Ry	on South wall's top	66,988	49,830	39,236	63,416
	Ry	on North wall's top	66,555	86,320	41,400	12,904

Tab. 5. *Reaction forces for the choir and nave structural units; Rx: horizontal reaction, Ry: vertical reaction. The number of frames and nodes is indicated in Figs. 4 and 5. OW: Own Weight; OW+W: Own Weight+Wind.*

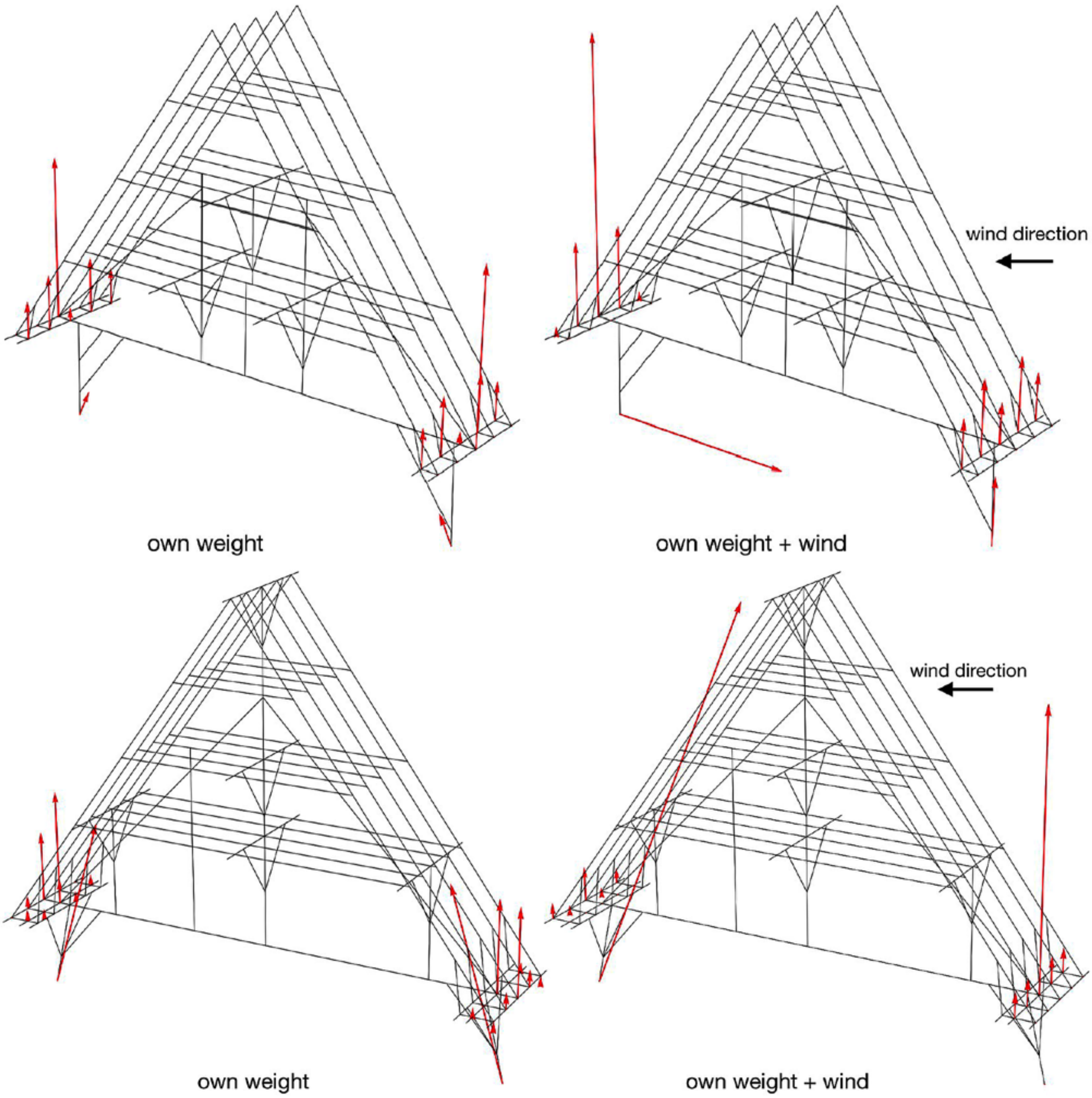

Fig. 7. *Distribution of the reaction forces in the original state; top, the choir's charpente; bottom, the nave's one.*

3.4.3. STRESSES

In Tabs. 8 and 9, the worst combination of internal actions, i.e., the one causing the highest stress value, is given for each different type of beam section, cf. Tab. 2. N is the axial force, positive when tension, M_1 and M_2 are the bending moments, T the shear force, σ_{max} a,d σ_{min} the highest and lowest values of the normal stress in the section, τmax the maximum of the shear stress. The wood being transversely isotropic, a strength criterion for anisotropic materials must be used to check whether or not the material is still in the elastic range [20]. Because wood has a brittle behavior and a different strength in tension and compression, the Hoffman criterion has been used for checking the strength of the structure [22].

By this criterion, for a plane state of stress, which in the case of bent beams, the matter is still in the elastic range if the failure index F_H, defined as

$$F_H = \frac{\sigma_{xx}^2}{X_t X_c} - \frac{\sigma_{xx}\sigma_{yy}}{X_t X_c} + \frac{\sigma_{yy}^2}{Y_t Y_c} - \frac{X_t - X_c}{X_t X_c}\sigma_{xx} - \frac{Y_t - Y_c}{Y_t Y_c}\sigma_{yy} + \frac{\sigma_{xy}^2}{S^2} \quad (1)$$

is not greater than one. In the case of beams, $\sigma_{yy} = 0$; moreover, in order to simplify the calculation, for each beam, we calculate F_H for $\sigma_{xx} = \{\sigma_{max}, \sigma_{min}\}$ and $\sigma_{xy} = \tau_{max}$ as reported in Tabs. 8 and 9, though, these stress values generally do not occur at the same point of the same beam. In this way, we obtain, for each beam, an upper bound F_H^{sup} for F_H, and the beam is in the elastic range if

$$F_H^{sup} = \frac{\sigma_{xx}^2}{X_t X_c} - \frac{X_t - X_c}{X_t X_c}\sigma_{xx} + \frac{\sigma_{xy}^2}{S^2} \leq 1. \qquad (2)$$

The characteristic values of X_t, X_c, and S have been chosen once more according to [18] and the European norm EN338, [19]: $X_t = 30$ MPa, $X_c = 29$ MPa, $S = 4$ MPa. The results for F_H^{sup} are shown in Tabs. 8, 9: $F_H^{sup} << 1$ in all the cases. In small words, the structure of both the *charpentes* is very feebly stressed: the matter is everywhere far below the elastic limit state. The most stressed pieces are the *jambes* on the leeward side under the action of the wind: $F_H^{sup} = 0.381$ for the choir's *charpente* and 0.496 for nave's one. The *entraits* are less stressed: the system of the intermediary supports is effective in reducing its bending.

	Weight of a SU	Load on the wall's top		Total wind force	
		South wall	North wall	Wx	Wy
Choir	143,561 35,015	66,988 16,338	66,555 16,233	37,020 9,029	4,900 1,195
Nave	141,956 40,559	39,236 11,210	41,400 11,829	30,260 8,646	4,304 1,230

Tab. 6. *Global loads on a structural unit (SU) of charpente, in [N]. In small: the values per unit length, in [N/m]. W_x and W_y are horizontal and vertical components of the total wind force.*

Node	Choir		Nave	
	δ_x	δ_y	δ_x	δ_y
	Own weight			
A	-0.17	-0.32	0.03	-0.51
B	-0.17	-1.27	0.04	-0.71
	Own weight+wind			
A	-16.60	-0.06	-7.35	-0.02
B	-16.43	-5.13	-7.12	-0.20

Tab. 7. *Displacements of points A and B in Fig. 6, [mm]; δ_x: horizontal displacement, δ_y: vertical displacement.*

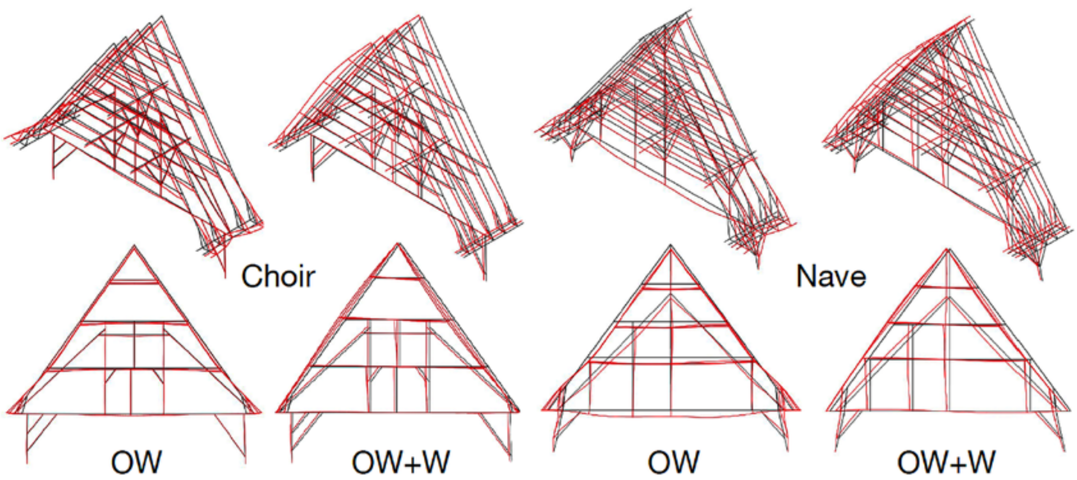

Fig. 8. *Deformation of the structural units of the charpentes, in the original state, for their own weight (OW) and for their own weight plus the wind (OW+W) (displacements magnified).*

	N [N]	M_1 [N m]	M_2 [N m]	T [N]	σ_{max} [MPa]	σ_{min} [MPa]	τ_{max} [MPa]	F^{sup}_H
				Own weight				
Entrait	37,422	4,590.06	0	1,915	1.100	-0.393	0.027	0.001
1st faux entrait	-2,453	2,111.70	0	4,298	1.267	-1.407	0.184	0.006
2nd faux entrait	-12,835	640.49	0	187	0.207	-0.982	0.008	0.002
3rd faux entrait	-2,958	1,231.29	0	772	0.840	-1.017	0.034	0.002
4th faux entrait	-1,774	177.97	0	0	0.135	-0.259	0.000	0.001
Arbalétriers	-8,103	1,666.67	0	104	1.302	-1.776	0.005	0.006
Faux arbalétriers	-24,633	629.24	0	1,346	-0.050	-0.984	0.042	0.002
Poteaux	14,368	2,345.56	0	9,240	3.700	-2.788	0.486	0.027
Poteau central haut	2,278	0.24	0	0	0.117	0.116	0.000	0.001
Jambe gauche, jambettes	-5,045	1,570.09	0	2,416	0.867	-1.111	0.088	0.003
Aiss. j. gauche, liernes	-6,988	91.72	0	0	-0.156	-0.399	0.000	0.001
Jambe droite	-4,934	837.54	0	2,323	0.377	-0.551	0.061	0.001
Sablières	0	1,805.65	3537.55	2,202	7.109	-7.109	0.124	0.067
Liernes	0	1,451.47	1.46	3,159	0.921	-0.921	0.135	0.003
				Own weight + wind				
Entrait	76,883	47,712.27	0	20,523	8.522	-7.058	0.293	0.079
1stfaux entrait	7,383	4,611.30	0	4,528	3.130	-2.709	0.194	0.014
2nd faux entrait	11,902	3,694.00	0	10,418	3.788	-3.070	0.471	0.028
3rd faux entrait	-6,224	5,359.04	0	4,839	3.872	-4.233	0.210	0.028
4th faux entrait	-1,991	177.97	0	0	0.120	-0.267	0.000	0.001
Arbalétriers	-2,712	1,345.38	0	777	1.163	-1.322	0.034	0.004
Faux arbalétriers	-29,961	1,578.62	0	873	0.541	-1.800	0.028	0.006
Poteaux	-9,436	4,148.06	0	2,499	5.491	-6.153	0.132	0.052
Poteau central haut	-2,113	266.43	0	148	0.475	-0.690	0.011	0.001
Jambe gauche, jambettes	83,028	24,063.63	0	27,403	17.160	-13.157	0.993	0.381
Aiss. j. gauche, liernes	105,058	91.72	0	0	4.290	4.048	0.000	0.016
Jambe droite	-12,232	131.55	0	363	-0.140	-0.287	0.010	0.001
Sablières	0	2,943.39	6302.16	3,589	12.224	-12.224	0.202	0.188
Liernes	0	620.64	4434.61	0	6.224	-6.224	0.000	0.052

Tab. 8. *Internal actions and stresses in the choir's charpente.*

	N [N]	M_1 [N m]	M_2 [N m]	T [N]	σ_{max} [MPa]	σ_{min} [MPa]	τ_{max} [MPa]	F^{Sup}_{H}
				Own weight				
Entrait	29,222	2,399.38	0	12,138	1.046	-0.271	0.241	0.008
Faux entraits	-5,704	1,107.67	0	4,323	0.539	-0.819	0.159	0.005
Arbalétriers	-20,504	937.66	0	2,374	0.038	-1.043	0.087	0.003
Faux arbalétriers	-18,252	441.47	0	2,169	-0.133	-0.997	0.101	0.004
Poinçon	17,586	404.86	0	3,803	0.707	0.102	0.131	0.002
Suspente	2,912	126.84	0	290	0.321	-0.119	0.015	0.001
Poteaux	6,788	259.42	0	896	0.429	-0.029	0.040	0.001
Jambettes	544	371.19	0	1,775	0.603	-0.557	0.111	0.002
Liernes	0	1,178.40	90.85	3,693	1.589	-1.589	0.205	0.011
Aisseliers and blochets	-8,124	50.04	0	0	-0.272	-0.450	0.000	0.001
Jambes de force	-31,443	1,777.49	0	14,055	1.322	-3.418	0.703	0.080
Chevrons secondaires	-6,009	1,096.50	0.02	248	0.525	-0.819	0.009	0.001
Sablières	0	0	728.81	0	0.630	-0.630	0.000	0.001
				Own weight + wind				
Entrait	41,322	12,805.76	0	12,138	4.062	-2.966	0.241	0.035
Faux entraits	9,001	5,752.51	0	4,323	3.745	-3.304	0.159	0.031
Arbalétriers	-37,182	4,388.64	0	2,374	1.620	-3.442	0.087	0.020
Faux arbalétriers	-48,625	2,562.83	0	2,169	1.000	-4.011	0.101	0.024
Poinçon	19,250	4,030.64	0	3,803	3.450	-2.564	0.131	0.022
Suspente	6,280	812.21	0	290	1.628	-1.192	0.015	0.004
Poteaux	-10,080	934.12	0	896	0.528	-1.121	0.040	0.003
Jambettes	1,170	866.57	0	1,775	1.403	-1.305	0.111	0.006
Liernes	0	237.72	3,198.68	0	5.032	-5.032	0.000	0.058
Aisseliers and blochets	-44,331	50.04	0	0	-1.881	-2.059	0.000	0.013
Jambes de force	-74,798	10,073.06	0	14,055	10.937	-15.924	0.703	0.496
Chevrons secondaires	-5,241	2,420.75	30.91	248	1.382	-1.638	0.009	0.006
Sablières	0	0	1,613.61	0	1.396	-1.396	0.000	0.004

Tab. 9. *Internal actions and stresses in the nave's charpente.*

3.5. ABOUT THE TRANSMISSION BY THE FRICTION OF THE HORIZONTAL FORCES

The results presented in the previous Sections have been found in the assumption that the horizontal forces cannot be taken up by friction. We ponder now on whether this assumption is or is not correct. To this end, a new simulation, considering the own weight uniquely, has been done, with changed boundary conditions: for each one of the nodes S1 to S10, a fixed support has been considered to simulate a perfect contact, i.e., a support with sufficient friction to stop any sliding. This is the condition implicitly understood when the horizontal thrust of the *charpente* on the top of the guttering walls is supposed to exist. The new simulation results are detailed in Tab. 10, where H is the total horizontal thrust of the *charpente* and V *is* the total vertical reaction for each structural unit applied to the top of the guttering walls.

	Choir		Nave	
	SW	NW	SW	NW
H [N]	46,048	45,356	40,790	43,389
V [N]	68,030	68,354	63,612	63,056
V_w [N]	160,000	160,000	136,100	136,100
V_t [N]	228,030	228,354	199,712	199,156
M [N m]	124,330	122,461	110,133	117,150
e [m]	0.545	0.536	0.551	0.588

Tab. 10. *Total forces, for each structural unit of charpente, on the top of the guttering walls in the assumption of fixed supports; H: horizontal load, V: vertical load, V_w: weight of the wall, V_t: total vertical load, M: overturning moment, e: eccentricity (NW: North wall; SW: South wall).*

We check the global equilibrium of the guttering wall 2.70 m below its top, i.e., at the same level as the corbels supporting the timber *consoles* of the *charpente*. This was the level of the top of the clerestory walls before the modifications started around 1220, and it is, to a good approximation, the free-standing height of the guttering walls. If we consider the thickness of the guttering walls of 60 cm and the density of the limestone of 2,400 kg/m^3, the weight V_w of this part of the guttering wall is ~160,000 N for the choir (length of 4.1 m) and of ~136,100 N for the nave (length of 3.5 m). The total vertical load V_t at the level -2.70 m with respect to the top of the wall can hence be calculated, as well as the overturning moment *M* of the horizontal thrust, and finally, the eccentricity *e* of V_t with respect to the centroid of the wall's section, cf. Tab. 10. The values of the eccentricity *e* so calculated, greater than 50 cm for all the cases, are extremely high and should cause the overturning of the wall. Also, if the wall had a greater thickness, the eccentricity should be too large to ensure a safe equilibrium of the system *charpente*-guttering walls.

We can hence check the physical possibility for the system to develop effective friction forces. The values of the horizontal, R_x, and vertical R_y, contact forces on the top of the guttering walls for the model with fixed bilateral supports are shown in Tab. 11. If we consider a friction coefficient ν = 0.7 for the contact between wood and stone, value usually admitted in such a case for a dry contact, then it appears clearly that the ratio R_x/R_y exceeds ν in several cases. At the same time, it is close to it in other cases. In addition, the contact between the wood of the *charpente* and the stone of the guttering walls is probably far from being perfect: infiltrations of dust and rainwater cannot be excluded, especially if one considers that the contact is actually unilateral and that, as shown in the numerical simulations presented in the section 3.4.1, some nodes of the *charpente* can slightly lift up under the action of the loads (cf. Fig. 8). It should also be mentioned that, as the same word *sablières* indicates (*sable* is the French word for sand), these wall plates were put in place over a layer of sand, to better ensure the contact between the *charpente* and the wall's top. Of course, this greatly decreases the friction forces, and the builders of the Middle Ages were likely aware of that.

Frame	Wall	Own weight			Own weight + wind		
		Rx [N]	*Ry* [N]	*Rx/Ry*	*Rx* [N]	*Ry* [N]	*Rx/Ry*
				Choir			
F1	North	22,058	26,586	0,83	30,368	31,337	0,97
	South	-21,342	26,604	0,80	-12,317	22,293	0,55
F2 and F3	North	5,983	10,378	0,58	8,933	10,650	0,84
	South	-5,994	10,339	0,58	-3,232	11,044	0,29
F4 and F5	North	6,012	10,506	0,57	9,254	11,570	0,80
	South	-6,013	10,374	0,58	-2,961	10,185	0,29
				Nave			
F1	North	14,523	21,722	0.67	26,051	28,120	0.93
	South	-15,064	22,122	0.68	-3,663	16,145	0.23
F2 and F3	North	8,119	10,281	0.79	9,216	13,483	0.68
	South	-6,490	10,338	0.63	-3,995	7,997	0.50
F4 and F5	North	6,314	10,386	0.61	9,137	13,796	0.66
	South	-6,373	10,407	0.61	-3,808	7,788	0.49

Tab. 11. *Reaction forces on the top of the guttering walls in the assumption of fixed supports; R_x: horizontal reaction, R_y: vertical reaction. For the frame numbering, cf. Fig. 5.*

Finally, the transmission, by friction, of horizontal forces between the *charpente* and the stone structure should be not only dangerous for the structure's safety but also rather uncertain or even impossible, physically speaking.

3.6. MODAL ANALYSIS OF THE *CHARPENTE*

In order to assess the difference in the structural behavior, namely in terms of stiffness, between the *charpentes* of the nave and of the choir, a modal analysis of the two *charpentes* has been performed. In fact, because the weight of the structural units of the two *charpentes* is practically the same (cf. Tab. 6), the higher the frequency, the higher the stiffness. For this analysis, the FE model is the same one used in the previous section for checking the friction mechanism. The first five modes of the choir's and nave's structures are presented in Fig. 9, where the corresponding frequencies are also indicated. What is apparent is that the nave's *charpente* has a greater stiffness than the choir's. In fact, if we compare the fundamental frequencies, mode 1, we can observe that the frequency of the nave's *charpente* is ~2.5 times that of the choir's one. This clearly indicates a better structural conception of the nave's *charpente* with respect to that of the choir.

4. ANALYSIS OF THE RESULTS AND THEIR INTERPRETATION

We now ponder the structural functioning of a *charpente à chevrons formant ferme*. We start with an analysis of the results of the structural study presented above. Then, on this basis, we try to draw some conclusions on the structural thought of the carpenters of the Gothic Age in France and, at the same time, compare these conclusions with the ideas and hypotheses existing in the literature to confirm or refute them.

4.1. THE MAIN RESULTS OF THE STRUCTURAL ANALYSES

The main points arising from the structural analyses presented above are:

- an intense concentration of the vertical reactions in correspondence with the main frames, while the vertical forces in correspondence with the supports of the common frames are much lower;
- the impossibility of transferring the horizontal forces from the charpente to the stone structure

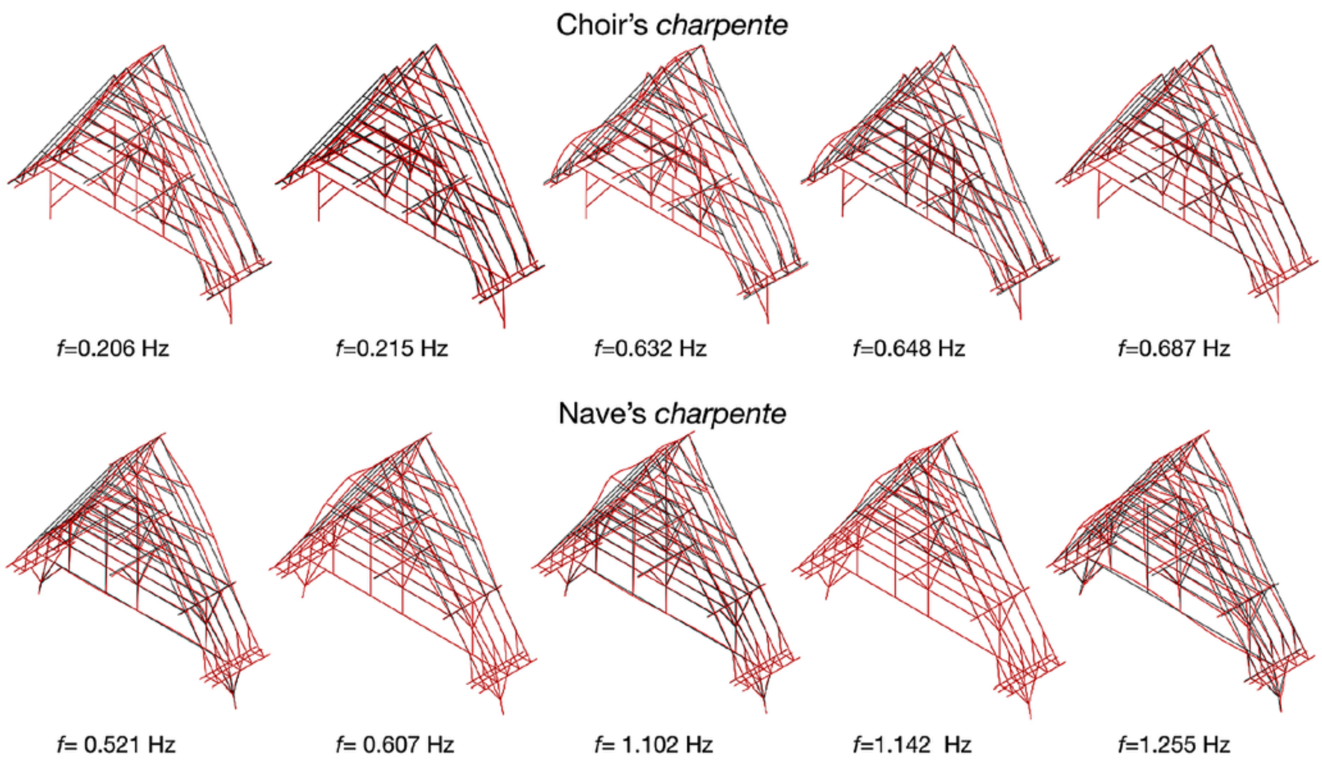

Fig. 9. *First five normal modes and corresponding vibration frequencies.*

below by friction and the fact that such a mechanism of transfer could cause the overturning of the guttering wall;
- the low levels of stress everywhere in the *charpente*;
- the slight deformation of the structure;
- the better design of the main frame of the nave with respect to that of the choir.

4.2. CRITICAL ANALYSIS OF THE RESULTS

Based on these results, the following conclusions can be drawn about the structural functioning of a *charpente à chevrons formant ferme*.

• **A three-dimensional structural functioning**: the structural behavior of a *charpente* with *chevrons formant fermes* is rather complicated and cannot be reduced to a simple planar scheme. The fundamental difference between the main and common frames is the tie-beam, used only for the main frames. Consequently, the horizontal thrust at the base of each secondary frame needs to be equilibrated differently; otherwise, the bending of the rafters would severely deform the frame. This is done by the system composed of the *sablières* and the *blochets*, see Fig. 3d. At the base of any rafter of a secondary frame, a *blochet* transfers the forces from the rafter to the *sablières*, restrained by the tie-beams of two successive main frames. The sablières are hence bent and sheared, in the horizontal plane, by the thrust at the base of the rafters of the secondary frames, and in this way, they transfer these horizontal thrusts, caused by the vertical loads, to the tie-beam of the main frames. Any modular structural unit of the *charpente* is hence self-equilibrated in the horizontal plane and does not apply any horizontal thrust to the stone structure below: a *charpente* with *chevrons formant ferme* is designed to transmit only vertical forces to the top of the guttering wall, while the horizontal thrusts engendered by the vertical loads are self-equilibrated by the system composed by the rafters, *blochets*, *sablières*, and tie-beams of the main frames. In particular, friction is not the mechanism of transfer of the horizontal thrust of the secondary frames to the top of the guttering walls.

In [23], some similar considerations are done, but with some differences. Viollet-le-Duc implicitly admits the role played by the system *blochets-sablières*, but he imputes its adoption for another reason. According to him, the invention of the *charpentes* with *chevrons formant ferme* with such important slopes served to adapt the roofing structure to the reduction of the thickness of the walls of the Gothic cathedrals, with respect to the Romanesque architecture, which rendered challenging to pose a *charpente* with main frames, purlins, and rafters. The only structural consideration made by Viollet-le-Duc concerns the high value of the slopes that decreases the bending of the rafters, so allowing the use of wood beams of relatively small cross sections (an important point, considered further): he never gives an explanation of the global functioning of the *charpente*, though it seems probable that he understood the danger of the transmission of the horizontal thrust from the common frames to the top of the guttering walls by friction.

The point of view of [24] is different: the increase of the roofs' slope in the Gothic period is essentially used to decrease the horizontal thrust. This interpretation cannot be considered correct, cf. Sect. 3.5. Also, the point of view of Pol Abraham [25], as reported in [3], is mechanically wrong: according to him, to explain the deformations of the Gothic vaults, he implicitly admits that there is a transfer, by friction, of the horizontal thrust of the *charpente* to the top of the guttering walls. The results presented above clearly indicate that this is not possible. The point of view of [3] is ambiguous: on the one hand, he strongly supports the idea of Viollet-le-Duc of the increase of the roofs' slope to use trunks of small diameters; on the other hand, he does not make any real structural consideration and tacitly, in the end, he seems to accept the point of view of Pol Abraham.

• **Distribution of the loads**: a common idea about the *charpentes* with *chevrons formant ferme* is that this structural scheme allowed an almost uniform distribution of the loads on the top of the guttering walls. As demonstrated in section 3.4.1, this is far from reality: the loads transferred by the charpente to the guttering walls are unevenly distributed, much higher in correspondence with the main frames than in the secondary ones. The reason for that is exactly the global structural organization that the carpenters gave to the *charpente*, i.e., its three-dimensional functioning.

Probably inspired by a rough two-dimensional analysis of the structure or also, perhaps, suggested by ideological positions, both of them far from the physical reality, the idea that the *chevrons formant fermes* were used to distribute the vertical load almost uniformly on the top of the clerestory walls does not correspond to reality. Moreover, we can affirm that the Middle Ages carpenters did not consider the correspondence of load-points for the *charpente* and the stone structure beneath, simply because they never coincide. In [26], also cited by [3], this discrepancy is severely criticized, based, on the one hand, upon a rather ideological point of view and, on the other hand, on a static idea that cannot be considered as valid, that of a structure that functions as an array of planar independent frames. In addition, the idea that the carpenters on one side and the masons of the stone structure on the other one worked separately, without interacting, is likely to be false: the stone corbels put at the base of the guttering walls exactly in correspondence with the main frames indicate that actually the wall construction of the cathedral was well planned and also conceived for the roofing structure.

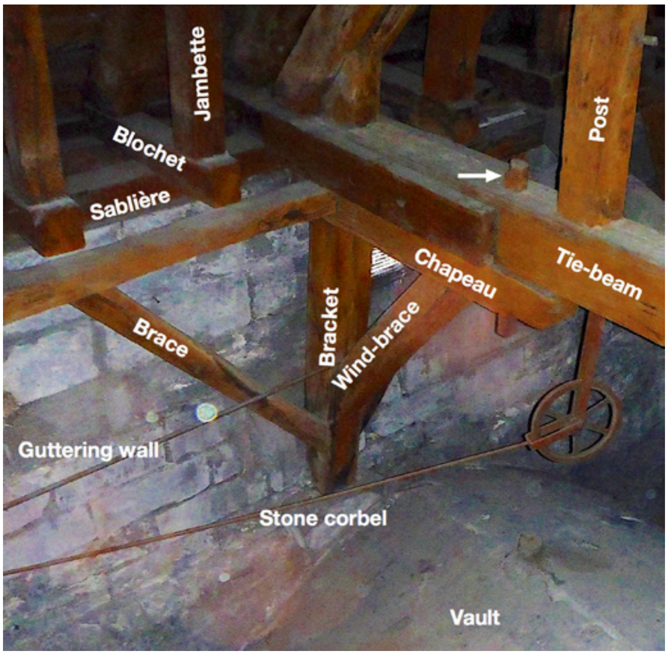

Fig. 10. *The system of the console for the transfer of the wind thrust to the guttering walls; the arrow indicates the shear key between the tie-beam and the chapeau of the console (by the Author).*

- **The wind thrust**: placed above such high constructions, the Gothic builders certainly did not ignore that the wind forces on the roof of a cathedral acted upon the stone structure below with a considerable thrust; also, as seen above, they knew that this thrust was impossible to be counterbalanced by friction, see section 3.5. There are few studies on how Gothic cathedrals, and their roofing structures, withstand the wind forces; namely, for what concerns Notre-Dame [14], with a historical perspective or [27] with nonlinear analysis.

The carpenters of Notre-Dame invented an effective system to transfer the wind action to the underlying stone structure: it is composed of the brackets (*jambes*) of the main frames with their windbraces (*aisseliers*) and *chapeau*, Fig. 10; in [9] this set of struts is indicated as the *console*. It is usually said that the *console* is used as a vertical support of the main frame or also for relieving the bending of the tie-beam. These ideas are not likely: the support given by the *sablières* is largely sufficient, and it should be easier and less expensive to use more posts for decreasing the bending of the tie-beam. In addition, observing the main frames of the nave, we can see that the *console* should support the *entrait* just where there is already a vertical tie, which is redundant. For the choir's frames, the span of the *entrait* is divided into three parts, which is largely sufficient to relieve it in bending, as confirmed by the numerical simulations presented in section 3.4.3. Actually, the true reason that led the carpenters of the 13th century to use the system of the *console* was to transfer the wind force to the lower part of the guttering wall: the horizontal thrust flows as an inclined force to the bottom of the guttering wall through the inclined *aisseliers* and the stone corbels close to the vault level, so improving the strength of the clerestory structure to the action of the wind considerably. This is also corroborated by the presence of a strong shear key connecting the tie-beam and the *console*, Fig. 10, whose role is to transfer the wind thrust from the tie-beam to the *console*, while it is completely useless for transmitting vertical forces. It is likely that the only reason that pushed the carpenters of the cathedral to introduce the *consoles* was to dispose of an excellent device to safely transmit the wind action on the roof to the stone structure below.

The system of the *consoles* is also useful for another reason: the *charpente* of a Gothic cathedral was, typically, built before the construction of the high vault, see, e.g. [3]. During this phase, the *charpente* was essential to counterbalance the flying-buttresses thrust, ensuring the connection between the two sides of the clerestory before the construction of the vault. The *consoles* well assumed this structural role: the tie-beams equipped with the two *consoles* could balance the inward thrust applied to the two opposite clerestory walls by the flying-buttresses. In the case of Notre-Dame, after the rise of the guttering wall, the top of the flying-buttresses was too far below the *charpente* to assume such a kind of thrust balance without a device, the *consoles*, acting down below the *charpente*.

• **The bracing system**: it is generally thought that a system composed of longitudinal beams (the girts) and braces placed at different levels was exclusively used by the carpenters as a bracing system. Indeed, the bracing system was important, especially during the constructive phases, to ensure the global stability of the *charpente* and, once built, to withstand longitudinal horizontal forces (though this is also provided, and more effectively, by the intersection of the nave/choir and transept *charpentes*; also the *voligeage*, acting as a sort of a plate connected to the frames, certainly contributed to the longitudinal stiffness of the structure). However, its true role was another one: the bracing system was used by the Gothic carpenters to relieve the bending of the collar ties (*faux entraits*) of the common frames and to transfer an important part of the vertical load from the secondary frames to the main ones. The numerical simulations (cf. section 3.4.1) confirm the effectiveness of the bracing system in transferring the vertical loads to the main frames, and, most importantly, it also helps in decreasing the bending in the rafters of the *fermettes* and the *sablières* because also the horizontal outward thrust at the base of the rafters is diminished by such a structural organization. Also, thanks to the bracing system, the carpenters did not need to use vertical ties in the *fermettes* to sustain the *faux entraits*. In short, the overall behavior of the *charpente*, conferred by the set of main and common frames and *sablières*, was insufficient to ensure the structure's equilibrium without the bracing system. There are three bracing systems in the choir of Notre-Dame's combles; in the nave, they are five (cf. Fig. 5).

• **The structural differences between the two *charpentes***: if the differences between the two *charpentes* are attentively considered, it appears that all the structural action of the nave's designer is oriented to increase the stiffness, and so the stability of the *charpente*: all the structural changes made with respect to the choir, i.e., the evolution of static scheme for the main frame, the reinforcement of the bracing system and the greater sections used for the *fermettes*, go in the direction of a stiffer structure. Actually, it is not the strength that is substantially improved because the level of stresses in the two *charpentes* is similar, Sect. 3.4.3, but the structure's stiffness, as confirmed by a comparative modal analysis, section 3.6. While a stress analysis was, without doubt, out of the means of the builders of the Middles Ages (the concept of stress was introduced by Cauchy in the 19th century [28]), an embryonic perception of the stability and hence of the stiffness, of a structure can have been in the abilities of the Gothic carpenters. It can be acquired through experience, especially during the construction phases. Thanks to this experience and ability, some particularly wise carpenters can have improved the technique, like in the case of Notre-Dame. Moreover, nowhere in the *combles* did the stresses reach important values: all the structure was scarcely solicited. Hence, all seem to indicate that the Gothics were mainly guided by increasing the structure's stiffness. Anyway, the change of the static scheme from the choir's *charpente* to the nave's one was certainly not dictated by economical reasons because the mass of wood for unit length along the longitudinal axis is greater for the nave.

• **The global structural functioning**: from what was said above, the global structural functioning of a *charpente* with *chevrons formant ferme* emerges clearly: the main frames collect and absorb, through the *sablières*, all the horizontal thrust caused by the vertical loads so that no horizontal action is conferred to the top of the guttering walls. Through the bracing system, the main frames also take on the part of the vertical load of the secondary frames that otherwise should be too much solicited in bending, like the *sablières*. The wind thrust is not transferred to the stone structure by friction on the wall's top, which could cause the tipping of these ones at their base, but by the system of the *consoles* and uniquely in correspondence of the main frames, the wind thrust is passed to the stone structure at the base of the wall on the leeward side as an inclined force. Along with the weight of the upper part of the wall, this allows the stone structure to withstand the action of the wind safely. Globally, the functioning of such a structure is three-dimensional; it is based upon strong collaboration and interaction of all the parts of the structure, and contrarily to what is often said, just like the underlying stone structure of the cathedral, it is a system that mainly transfers the loads at some points, in correspondence of the main frames, and not continuously. On the whole, we can today appreciate the ability of the carpenters of the Middle Ages, that were capable of inventing a very effective structural system well before the discoveries of structural mechanics; such timber structures have spanned the centuries, which witnessed their effectiveness, their only and true danger seems to be the neglect of humans.

A question remains: for what reasons did the Gothics invent the *charpentes* with *chevrons formant ferme*? The following section is devoted to answering this question.

5. CONCLUSION: WHY THE *CHARPENTES À CHEVRONS FORMANT FERME*?

The invention of the *charpentes à chevrons formant ferme* actually responds to an evolution of Gothic architecture, evident just in the case of Notre-Dame [23]: the rise of the guttering walls above the top of the vault. Before such a modification, it was not possible to use frames with tie-beams, because the vault's top was higher than the footing of the frames. The solution adopted by the carpenters was probably that of scissor trusses that do not have a tie-beam. We do not know what the reason for the raising of the guttering walls was, a solution later adopted also in other cathedrals; what is certain is that the carpenters of the 13th century could then adopt a static scheme different from the scissor truss, but they were also faced to the problem of scarcity of trunk of sufficient dimensions (it is famous the adventurous pilgrimage of Abbot Suger to find trees of adequate size for the roof of the Royal Abbey of Saint-Denis), see [3].

This was the true problem for the carpenters of the period: to dispose of wooden pieces of sufficiently large dimensions. In particular, the use of long *entraits* (in Notre-Dame, they have a length of 13 m) must dispose of beams of a significant section to withstand the rod's bending. This was not so easy in the France of the period, and the use of a *charpente* with few *entraits* was hence almost compulsory. So, to solve a structural problem (efficient roofing structures of large dimensions) with the scarcity of sufficiently large timber beams, the carpenters of the beginning of the 13th century invented the *charpente* with *chevrons formant ferme*. Other great cathedrals were covered with this type of structure, among the still-existing ones Amiens [29] and Bourges [30]. With this solution, they brilliantly solved not only the problem of scarcity of sufficiently large wooden beams but transformed the statical problem, adopting a solution that allows eliminating the horizontal thrust at the top of the guttering walls, which are too high to withstand such forces. This solution witnessed a radical change in the structural thought of the carpenters of the period: they were able to pass from a bi- to a three-dimensional scheme, where all the parts of the structure interact together. In the end, we can appreciate how deep and subtle was the structural knowledge of the master builders of the Middle Ages.

6. REFERENCES

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